2009 Paper 6 Question 5

Foundations of Functional Programming

(*a*) Define the Church numerals giving the encodings of zero $\underline{0}$, one $\underline{1}$ and an arbitrary number $\underline{n}$. [3 marks]

(*b*) Define λ-terms to perform the following operations on Church numerals. You may assume standard definitions for Booleans (true, false, if, and, and or) and pairs (pair, fst, and snd). For each part, you may assume solutions to the previous parts of the question. You may *not* use a fixed-point combinator.

(*i*) Test for zero. [2 marks]

(*ii*) Successor. [2 marks]

(*iii*) Predecessor (where predecessor of zero is zero). [4 marks]

(*iv*) Less than or equal. [3 marks]

(*v*) Equality. [2 marks]

(*vi*) Successor modulus n (where $\mathsf{succn}\,\underline{n}\,\underline{m} = \underline{0}$ if $n = m + 1$, and $\mathsf{succn}\,\underline{n}\,\underline{m} = \underline{m+1}$ otherwise). [2 marks]

(*vii*) Modulus (e.g $\mathsf{mod}\,\underline{n}\,\underline{m} = \underline{m \bmod n}$). [2 marks]