2009 Paper 8 Question 12

Specification and Verification I

(*a*) What is *Russell's Paradox*? Explain how it is avoided in higher-order logic.
[4 marks]

(*b*) Explain how the universal quantifier $\forall$ is defined in higher-order logic. Write down a definition of the quantifier and give its type. [4 marks]

(*c*) Define a constant ExistsFour in higher-order logic such that ExistsFour P represents "$P(x)$ *is true for at least* 4 *values of* x". [4 marks]

(*d*) Define a constant TotalSpec in higher-order logic that represents the total correctness specification $[P]\ C\ [Q]$. What are the types of the terms representing P, C and Q? [4 marks]

(*e*) Using TotalSpec from your answer to part (*d*), write down formulae that represent the sequencing and WHILE rules for total correctness. [4 marks]