2009 Paper 8 Question 3

Computer Systems Modelling

(*a*) Suppose that X is a random variable having the Binomial distribution with parameters n and p and that $\lambda > 0$ is a constant.

(*i*) Write down the expression for $\mathbb{P}(X = k)$ where $k \in \{0, 1, 2, \ldots, n\}$ [2 marks]

(*ii*) Now suppose that $n \to \infty$ and p is chosen so that $p = \lambda/n$. Show that under this limit $\mathbb{P}(X = k) \to e^{-\lambda}\lambda^k/k!$, that is, to a Poisson distribution with parameter λ. [4 marks]

(*b*) Suppose that $N(t)$ is the random number of events in the time interval $[0, t]$ of a Poisson process with parameter λ.

(*i*) State the conditions that define the Poisson process $N(t)$. [2 marks]

(*ii*) Show that for all $t > 0$ the random variable $N(t)$ has the Poisson distribution with parameter λt. [4 marks]

(*c*) Given a Poisson process of rate λ let X_1 be the time of the first event and for $n > 1$ let X_n denote the time between the events $(n - 1)$ and n. Thus the sequence $X_1, X_2, \ldots$ gives us the sequence of *inter-event times* between the events in a Poisson process.

(*i*) Show that

$$\mathbb{P}(X_1 > t) = \mathbb{P}(N(t) = 0)$$

for $t > 0$. [4 marks]

(*ii*) Show that the inter-event times $X_1, X_2, \ldots$ are independent, identically distributed random variables each of whose marginal distribution is an Exponential with rate parameter λ. [4 marks]