2010 Paper 3 Question 1

Algorithms II

(*a*) Describe Dijkstra's shortest-path algorithm, making the priority queue operations explicit. [4 marks]

(*b*) Provide a small example demonstrating that Dijkstra's shortest-path algorithm will not work correctly when negative weights are used on some arcs.
[3 marks]

(*c*) Suppose some arcs in a directed graph have negative weights, and that $-W$ is the least negative weight among all arcs. Suppose that we add W to all arcs in the graph to obtain a new graph with non-negative arc weights. Will the resulting graph have the same shortest paths as the original graph? Explain your answer. [3 marks]

(*d*) For each of the data structures listed below, describe the computational complexity of Dijkstra's shortest-path algorithm when this data structure is used to implement the algorithm's priority queue. Justify your answers.

    (*i*)   An unsorted array, indexed by node number. [2 marks]

    (*ii*) A linked list, sorted by key (in this case a distance estimate). [2 marks]

    (*iii*) A binary heap. [2 marks]

    (*iv*) A binomial heap. [2 marks]

    (*v*)   A Fibonacci heap. [2 marks]