2010 Paper 6 Question 3

Computation Theory

(a) Define the notion of a *register machine* and the computation it carries out. [5 marks]

(b) What does it mean for a partial function $f(x_1, \ldots, x_n)$ of n arguments to be *register machine computable*? [3 marks]

(c) Why do there exist partial functions that are not register machine computable? (Any standard results you use in your answer should be carefully stated.) [3 marks]

(d) Consider the following register machine program.

$$\begin{array}{l} L_0 : R_1^- \rightarrow L_1, L_6 \\ L_1 : R_2^- \rightarrow L_2, L_4 \\ L_2 : R_0^+ \rightarrow L_3 \\ L_3 : R_3^+ \rightarrow L_1 \\ L_4 : R_3^- \rightarrow L_5, L_0 \\ L_5 : R_2^+ \rightarrow L_4 \\ L_6 : \text{HALT} \end{array}$$

Assuming the contents of registers R_0 and R_3 are initially zero, what function of the initial contents of registers R_1 and R_2 does this program compute in register R_0 upon halting? (You may find it helpful to consider the graphical representation of the program.) [4 marks]

(e) Let $f(x_1, x_2)$ be the partial function that is equal to $x_1 - x_2$ if $x_1 \geq x_2$ and is undefined otherwise. Give a register machine program that computes f. [5 marks]