2010 Paper 7 Question 4

Artificial Intelligence II

Professor Elbow-Patch is not the man he used to be, and in particular has a tendency to fall over for no apparent reason. This problem is made worse if he has drunk port with his dinner. He almost always drinks port on a Sunday, and if he drinks on any given day he is unlikely—for the sake of his long-suffering liver—to drink port on the following day. However, if he does not drink on a given day then he is very likely to succumb to temptation on the following day.

The probability that he falls over after drinking is $\Pr(\text{fall}|\text{drank}) = 0.7$. The probability that he falls over when he has not drunk is $\Pr(\text{fall}|\neg\text{drank}) = 0.1$. He drinks on a Sunday with probability 0.9. If he has not drunk on a given day then the probability that he drinks the following day is $\Pr(\text{drink today}|\neg\text{drank yesterday}) = 0.8$. If he has drunk on a given day then the probability that he drinks the following day is $\Pr(\text{drink today}|\text{drank yesterday}) = 0.1$.

(*a*) Explain how this problem can be represented as a *hidden Markov model*. What assumptions are required? [4 marks]

(*b*) Denoting observations at time i by E_i and states at time i by S_i give a derivation of the *filtering algorithm* for computing $\Pr(S_t|E_1, \ldots, E_t)$. [8 marks]

(*c*) You observe the Professor on Sunday, Monday and Tuesday and notice that he does not fall over at all. Use the filtering algorithm to compute the probability that he drank port on Tuesday. [8 marks]