

2011 Paper 1 Question 5

Algorithms I

Mathematical hint: the following series converges to the indicated value if $|x| < 1$

$$\sum_{m=1}^{\infty} mx^m = \frac{x}{(1-x)^2}$$

- (a) The binary min-heap provides a `decreaseKey()` method that, when applied to an element, decreases its key while preserving the properties of the data structure.
- (i) Give a brief and clear description of how `decreaseKey()` works. [3 marks]
- (ii) The standard `decreaseKey()` accepts only a positive argument. Describe an implementation that removes that restriction, that is to say a heap method that can move the key value up as well as down, as specified by the sign of its argument. [4 marks]
- (b) Generalise the *binary* min-heap to one where nodes have not 2 but k children.
- (i) State the two defining properties of a min-heap, one constraining the shape and one constraining the keys of the data structure, and describe how to represent a k -ary min-heap as an array. [4 marks]
- (ii) Give a clear description of an algorithm (a simple generalisation of the well-known one for binary heaps) that takes an arbitrary n -item array and efficiently rearranges its elements to turn it into an array representing a k -ary heap. [4 marks]
- (iii) Analyse its complexity as a function of n and k . [5 marks]