

2011 Paper 1 Question 6

Algorithms I

Mathematical hint: the following series converges to the indicated value if $|x| < 1$

$$\sum_{m=1}^{\infty} mx^m = \frac{x}{(1-x)^2}$$

- (a) You are given an unsorted array of n items of which you must return the top k , in any order. Give a clear and accurate description of two efficient algorithms for solving the problem, following the hints below, and derive their time complexity in terms of n and k .
- (i) For this attempt, use a priority queue. [2 marks]
- (ii) For this attempt, start by finding the k -th largest item, as when computing order statistics. Describe all the steps in detail. [8 marks]
- (b) You are given a hash table that stores n keys. It has m slots; collisions are resolved by chaining (each hash table slot contains a pointer to the head of the corresponding chain; no elements are stored in the table itself) and no chain is longer than L elements. Each slot has an extra field giving the length of the corresponding chain.

Give a clear and accurate description of an efficient algorithm for returning a random key from the table, such that each key has equal probability of being selected, and analyse its time complexity. Assume the availability of a function `random(x)` that returns in constant time a random integer between 0 and x .

The time complexity of a reasonable algorithm will not exceed $O(m + L)$, but solutions awarded full marks will improve on that.

[10 marks]