2011 Paper 6 Question 2

Complexity Theory

(a) Recall that a Boolean formula is in 3-CNF if it is the conjunction of clauses, each of which is the disjunction of *at most three* literals. A literal is either a variable or a negated variable.

Consider the following two decision problems:

> 3-SAT: given a Boolean formula in 3-CNF, decide whether or not it is *satisfiable*.
>
> 3-VAL: given a Boolean formula in 3-CNF, decide whether or not it is *valid*.

(i) One of the two problems above is known to be in the complexity class **P**. Which one, and why? [2 marks]

(ii) Describe a polynomial time algorithm for the problem you identified in part (i). [6 marks]

(iii) What can you say about the complexity of the other problem? State precisely any standard results you use in your answer. [4 marks]

(b) Say that a Boolean formula is in 3-DNF if it is the disjunction of terms, each of which is the conjunction of *at most three* literals.

We now consider the following two decision problems:

> 3-DNF-SAT: given a Boolean formula in 3-DNF, decide whether or not it is *satisfiable*.
>
> 3-DNF-VAL: given a Boolean formula in 3-DNF, decide whether or not it is *valid*.

What can you conclude about the complexity of these two problems? [8 marks]