2011 Paper 6 Question 7

Mathematical Methods for Computer Science

(a) Let X be a random variable with finite mean, $E(X)$, and variance, $\text{Var}(X)$, and let $a > 0$.

    (i) Show Markov's inequality that $P(|X| \geq a) \leq \frac{E(|X|)}{a}$.   [5 marks]

    (ii) Using Markov's inequality show that $P(|X| \geq a) \leq \frac{E(X^2)}{a^2}$.   [5 marks]

(b) A study by a mobile phone operator shows that the expected number of simultaneous calls at a base station is 100. The actual number of simultaneous calls is a random variable, X, and so the base station is designed to handle a higher number of simultaneous calls up to a maximum of $M = 150$.

    (i) Use the Markov inequality to bound the probability that the station will receive more than 150 calls.   [5 marks]

    (ii) Now suppose that we are given the additional information that the variance of the number of simultaneous calls is 50. Use the inequality from part $(a)(ii)$ to give a second bound on the probability of exceeding 150 calls.   [5 marks]