2 Complexity Theory (AD)

(*a*) Consider the following decision problem.

> Given positive integers $x_1, \ldots, x_n, y$, determine whether there is a set $I \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in I} x_i = y$.

(*i*) Prove that, if the integers $x_1, \ldots, x_n, y$ are written in *unary*, then the problem is in **P**. [*Hint:* consider a recursive algorithm which checks whether either of y or $y - x_1$ can be expressed as the sum of a subset of $x_2, \ldots, x_n$.] [6 marks]

(*ii*) What can you say about the complexity of the decision problem when the integers $x_1, \ldots, x_n, y$ are written in *binary*? You do not need to prove your answer, but state clearly any standard results you use. [2 marks]

(*b*) What does it mean for a language L to be **NP**-hard? What does it mean for L to be **NP**-complete? [2 marks]

(*c*) We write $[M]$ to be the string encoding a Turing machine M using some standard coding scheme. Consider the language A defined by:

$$A = \{[M], x \mid M \text{ accepts } x\}$$

where "$[M], x$" denotes the string $[M]$ followed by a comma and then x.

(*i*) Prove that A is **NP**-hard. [8 marks]

(*ii*) Is A **NP**-complete? Justify your answer. [2 marks]