COMPUTER SCIENCE TRIPOS Part II – 2012 – Paper 9

2 Computer Systems Modelling (RJG)

(a) Consider a general birth-death process with birth rates λ_i and death rates μ_i in state i $(i = 0, 1, 2, \ldots)$. What are the detailed balance equations for this process? [2 marks]

(b) Derive the steady-state distribution for the general birth-death process considered in part (a). What are the conditions for the steady-state distribution to exist? [4 marks]

(c) Describe the $M/M/1$ queue and give a stochastic model for the number N of customers present. Find the steady-state distribution for N and state the conditions for it to exist. [4 marks]

(d) Derive the mean and variance of N. [4 marks]

(e) State Little's law and use it to derive the mean time spent in the $M/M/1$ queue under steady state conditions. [2 marks]

(f) Discuss what is meant by the traffic intensity for an $M/M/1$ queue and explain what happens to the distribution of the number of customers present as the traffic intensity increases towards one. [4 marks]