

5 Logic and Proof (LCP)

(a) In the context of clause methods in theorem proving, define and discuss the concept of a *pure literal*. [3 marks]

(b) Use the DPLL method to find a model satisfying the following set of formulas or to prove that no such model exists.

$$\begin{aligned}(P \wedge R) &\rightarrow Q \\ \neg(P \wedge Q \wedge R) & \\ (R \vee \neg Q) &\rightarrow P \\ P &\rightarrow R\end{aligned}$$

[5 marks]

(c) For each of the following formulas, either exhibit a formal proof (in a sequent or tableau calculus) or exhibit a falsifying interpretation.

(i)

$$\forall x(P(x) \rightarrow Q(x)) \rightarrow (\exists xP(x) \rightarrow \exists xQ(x))$$

[6 marks]

(ii)

$$\exists x(P(x) \rightarrow Q(x)) \rightarrow (\forall xP(x) \rightarrow \forall xQ(x))$$

[6 marks]