**COMPUTER SCIENCE TRIPOS Part IA – 2015 – Paper 1**

## 5 Numerical Methods (DJG)

(*a*) Consider the iteration:

$$x_{n+1} = (2x_n + N/x_n^2)/3$$

(*i*) The iteration converges to give what useful property of the constant argument $N$? [2 marks]

(*ii*) Examine whether the above iteration should work for all possible values of $N$ and $x_0$? [6 marks]

(*iii*) Find the order of convergence for the above iteration. You may use standard results but do not simply state an order without justification. [3 marks]

(*b*) Cholesky provides an approach to solving certain systems of simultaneous equations. His method (and similar methods) perform upper/lower triangle decomposition of the equation coefficient matrix $A$ such that $A = LU$ and $U^T = L$.

(*i*) Under what circumstances can Cholesky's method be used? Can it be used if $A$ is already a triangular matrix and, if not, what should be done instead? [3 marks]

(*ii*) Give expressions for two of the four values in the upper-left $2 \times 2$ sub-matrix of $L$ in terms of the elements of $A$. [2 marks]

(*iii*) When is Cholesky's method preferred over general Gaussian Elimination and what advantage does it provide? [3 marks]

(*iv*) Why might the decomposition $A = LDU$ be preferable to an $LU$ decompostion given that $D$ is a diagonal matrix? [1 mark]