3 Computation Theory (AMP)

(*a*) What does it mean for a partial function to be *register machine computable*?

[3 marks]

(*b*) Give definitions of bijective codings of pairs of numbers $(x, y) \in \mathbb{N}^2$ as numbers $\langle x, y\rangle \in \mathbb{N}$; and of finite lists of numbers $\ell \in \mathit{list}\,\mathbb{N}$ as numbers $\ulcorner \ell \urcorner \in \mathbb{N}$.

[3 marks]

(*c*) Let T be the subset of $\mathbb{N}^3$ consisting of all triples $(e, \ulcorner[x_1, x_2, \ldots, x_m]\urcorner, t)$ such that the computation of the register machine with index e halts after t steps when started with $\mathtt{R}_0 = 0, \mathtt{R}_1 = x_1, \ldots, \mathtt{R}_m = x_m$ and all other registers zeroed. Define a function $s \in \mathbb{N}{\to}\mathbb{N}$ as follows. For each $n \in \mathbb{N}$, $s(n) \in \mathbb{N}$ is the maximum of the finite set of numbers $\{t \mid \exists e, x \in \mathbb{N}.\ \langle e, x\rangle \leq n \ \wedge\ (e, x, t) \in T\}$.

Prove that for all recursive functions $r \in \mathbb{N}{\to}\mathbb{N}$, there exists some $n \in \mathbb{N}$ with $r(n) < s(n)$. Any standard results about register machines and about recursive functions that you use should be clearly stated, but need not be proved.

[14 marks]