COMPUTER SCIENCE TRIPOS Part IB – 2016 – Paper 6

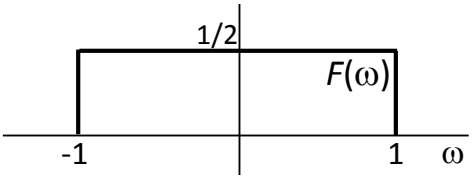
7 Mathematical Methods for Computer Science (JGD)

(*a*) (*i*) Express W, the primitive N^{th} root of unity, as a complex exponential. [2 marks]

(*ii*) Express the N-point real-valued discrete sequence $f[n] = \cos(2\pi n/N)$ for $n = 1, 2, 3, \ldots, N$ in terms of W. [3 marks]

(*iii*) Using a vector sum diagram in the complex plane, show how elements of the real-valued discrete sequence $f[n]$ are represented as a sum of complex numbers related to W, each having unit length. Construct your diagram for the particular case of integer $n = N/8$. [2 marks]

(*b*) A zero-centred pulse function $F(\omega)$ in the frequency domain ω, having unit area $F(\omega) = 1/2$ for $\omega \in [-1, +1]$, and $F(\omega) = 0$ for $\mid \omega \mid > 1$, represents one ideal low-pass filter.



(*i*) Derive its inverse Fourier transform $f(x)$. [4 marks]

(*ii*) Sketch a plot of this function and specify the roots of $f(x) = 0$. [2 marks]

(*c*) Let $f(x)$ be any real-valued function whose Fourier transform $F(\omega)$ exists. Show that $F(\omega)$ has the property of Hermitian symmetry $F(-\omega) = \overline{F(\omega)}$, and comment on the computational benefits that result from this property.

Hint: Represent $f(x)$ as the sum of an even function $f_e(x)$ plus an odd function $f_o(x)$, where

$$f_e(x) = \frac{1}{2}(f(x) + f(-x))$$
$$f_o(x) = \frac{1}{2}(f(x) - f(-x))$$

and then consider the Fourier transform of $f(x) = f_e(x) + f_o(x)$. You may invoke known properties of even- and odd-symmetric functions without proof. [7 marks]