4 Computer Systems Modelling (RJG)

(*a*) (*i*) Suppose that $F_X(x)$ is a distribution function. Show the *inverse transform result*, namely that, if U is a random variable uniformly distributed in the interval $(0, 1)$ then

$$X = F_X^{-1}(U)$$

is a random variable with distribution function $\mathbb{P}(X \le x) = F_X(x)$.
[4 marks]

(*ii*) Discuss the notion of a pseudo-random number generator for uniform random variables. Describe suitable algorithms for generating pseudo-random numbers. [6 marks]

(*iii*) Using the inverse transform result in part (*a*)(*i*) derive a method to generate a stream of independent pseudo-random numbers from an exponential distribution with parameter $\lambda > 0$. What are the true mean and variance of these numbers in terms of λ? [4 marks]

(*b*) (*i*) Suppose that you conduct a simulation experiment to estimate the mean, μ, of a random quantity X from a sample of n values $X_1, X_2, \ldots, X_n$. How would you estimate μ? [2 marks]

(*ii*) Now suppose that your simulation also yields a sample of n values $Y_1, Y_2, \ldots, Y_n$ of the random quantity Y where $\mathbb{E}(Y) = \mu_Y$ is a known number. How would you use the method of *control variates* to improve your estimator of μ? Your answer should mention all quantities that may need to be estimated and in what way you will improve the estimation of μ.
[4 marks]