5 Logic and Proof (LCP)

(*a*) Exhibit an interpretation in S4 modal logic that simultaneously satisfies the formulas $P \wedge Q$, $\Box(P \vee Q)$, $\Diamond\neg P$, $\Diamond\neg Q$ at a particular world, w. [5 marks]

(*b*) For each of the following sets of clauses, either exhibit a model or show that none exists. Below, a and b are constants, while x, y and z are variables. Briefly justify your answers.

(*i*)

$$\{\neg R(x,y), R(f(x),f(y))\}$$
$$\{R(a,b)\} \quad \{\neg R(x,x)\}$$
$$\{\neg R(y,x), R(y,z), \neg R(x,z)\}$$

[7 marks]

(*ii*)

$$\{\neg Q(x,y), \neg Q(y,x), R(x)\}$$
$$\{\neg P(a,y), Q(y,y)\}$$
$$\{\neg Q(x,y), P(b,x)\}$$
$$\{P(z,b), P(x,y)\}$$
$$\{\neg R(b), \neg R(y)\}$$

[8 marks]