

6 Logic and Proof (LCP)

- (a) Draw ordered binary decision diagrams (BDDs) for each of the following formulas, thereby identifying which of them are logically equivalent.

$$\begin{aligned} P &\rightarrow (Q \rightarrow R) \\ P &\rightarrow (R \rightarrow Q) \\ (\neg Q \vee R) &\vee \neg P \end{aligned}$$

[8 marks]

- (b) A mysterious propositional connective,  $\odot$ , has the following right-side sequent calculus rule,  $(\odot r)$ :

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, A \odot B}$$

Present the corresponding left-side sequent calculus rule,  $(\odot l)$ , along with the truth table for  $\odot$ . [6 marks]

- (c) For the following formula, either exhibit a formal proof (using the sequent calculus, augmented with the  $(\odot r)$  rule above) or exhibit a falsifying interpretation:

$$\Rightarrow \exists x(P(x) \odot Q(x)), (\forall x P(x)) \wedge (\forall x Q(x))$$

[6 marks]