## 10 Quantum Computing (AD)

Recall that the four states of the *Bell basis* are:

$$\beta_{00} = \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \qquad \beta_{01} = \tfrac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$\beta_{10} = \tfrac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \qquad \beta_{11} = \tfrac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

(*a*) Give, in matrix form, a unitary operator $B$ such that for any computational basis state $|ij\rangle(i, j \in \{0,1\})$, we have $B|ij\rangle = |\beta_{ij}\rangle$. [2 marks]

(*b*) For the operator $B$ defined in part (*a*), give a description of the six states: $B^2|00\rangle$, $B^2|01\rangle$, $B^3|01\rangle$, $B^3|10\rangle$, $B^4|11\rangle$, $B^4|10\rangle$. [6 marks]

(*c*) Consider a *quantum finite automaton* in a two letter alphabet $\{a, b\}$. The automaton has four states: $|0\rangle, |1\rangle, |2\rangle, |3\rangle$. The transitions on input $a$ are given by the rule:

$$M_a : |i\rangle \mapsto |(i+1) \bmod 4\rangle$$

and the transition matrix for input $b$ is given by the operator $B$ from part (*a*).

For each of the strings $w$ below, give the probability that the automaton, when started in state $|0\rangle$, after reading $w$ is measured to be in state $|0\rangle$.

(*i*) $aaa$

(*ii*) $a^8$

(*iii*) $a^3b^4$

(*iv*) $ababab$

[2 marks each]

(*d*) For the automaton defined in part (*c*), give a complete description of all strings $w$ such that when the automaton is started in state $|0\rangle$ it reaches state $|0\rangle$ with probability 1 after reading $w$. [4 marks]