

4 Computer Systems Modelling (IML)

A system is to be built that can serve customers at a rate μ . Three different configurations, A , B , and C , are to be considered:

A has two servers each serving at a rate $\mu/2$, each with its own queue,

B has two servers each serving at a rate $\mu/2$, but with a shared queue,

C has one server serving at a rate μ and has a single queue,

Customers arrive into the system at a rate λ . In the case of configuration A , customers join the shortest queue and cannot change queues subsequently. If the queues are the same length, the choice is random with equal probability. The arrival process is Poisson and service times are exponentially distributed.

- (a) Which two of these can be modelled as *birth-death* processes? [2 marks]
- (b) Draw state transition diagrams representing corresponding Markov processes for each option. Your diagrams need not go beyond four customers in the system. [4 marks]
- (c) For each of the birth-death processes, derive the steady state probabilities, expected utilisation and average queue length. [10 marks]
- (d) Which of the birth-death configurations gives the shorter expected time in the system for a customer? Under what circumstances is this most pronounced? [3 marks]
- (e) Consider the configuration that is not a birth-death process. What small change could be made that would make it easier to analyse? [1 mark]