10 Logic and Proof (lp15)

(*a*) For each of the following formulas, present either a formal resolution proof or a falsifying interpretation. Note that a and b are constants.

$$\forall x[Q(x) \to R(x)] \land \neg R(a) \land \forall x[\neg R(x) \land \neg Q(x) \to P(b) \lor Q(b)] \to P(b) \lor R(b)$$

[4 marks]

$$\exists x[\forall yz[(P(y) \to Q(z)) \to (P(x) \to Q(x))]]$$

[4 marks]

(*b*) For each of the following formulas, present a proof in a sequent or tableau calculus, or alternatively, a falsifying interpretation. In Part (*b*)(*iii*) the modal logic is S4.

(*i*) $\exists y\, \forall x\, P(x, y) \to \exists z\, P(z, z)$ [3 marks]

(*ii*) $\forall x\, [P(x) \land \exists y\, \neg P(y)] \to Q$ [5 marks]

(*iii*) $(\Box\Diamond P \land \Box\Diamond Q) \to \Box\Diamond(P \land Q)$ [4 marks]