14 Quantum Computing (sjh227)

(*a*) A classical bit-flip channel has probability of error p, and a n-bit repetition code is used to suppress the error. If n is even, find the probability that a 'majority vote' decoding returns no answer. [2 marks]

(*b*) A qubit is encoded using a 3-bit repetition code. If it is known that the qubits will only ever encounter noise that can be modelled as independent, identically distributed bit-flips, with the probability of a bit flipping equal to p, then give the threshold of this code. State any assumptions made. [3 marks]

(*c*) A certain error-correction code suppresses the physical qubit error, p, to $\mathcal{O}(p^2)$ and has a threshold of 1%. For a quantum circuit with 20 gates, find the number of layers of concatenation required to achieve an overall error probability of at most 10% when:

(*i*) The gate error-rate is 0.99%.

(*ii*) The gate error-rate is 0.9%.

[5 marks]

(*d*) For a certain implementation of a 3-qubit phase-flip code the *principle of deferred measurement* is invoked to allow the recovery operations to be enacted conditional on qubit states rather than measurement outcomes. Let $|m\rangle$ be the two-qubit state of the parity check qubits, then the recovery circuit must perform the following operations on the three code qubits:

$\|m\rangle$	**Recovery Operations**
$\|00\rangle$	$I \otimes I \otimes I$
$\|10\rangle$	$Z \otimes I \otimes I$
$\|11\rangle$	$I \otimes Z \otimes I$
$\|01\rangle$	$I \otimes I \otimes Z$

Design the recovery circuit using only gates from the set: $\{H, T, \text{CNOT}, \text{Toffoli}\}$. [6 marks]

(*e*) How many more gates would be required if only gates from the set $\{H$, T, CNOT$\}$ can be used in the recovery circuit for Part (*d*)? [4 marks]