6 Introduction to Probability (tms41)

(*a*) A laptop is expected to run for X number of hours from new until it breaks down. Let X be an exponential random variable with $\lambda = \frac{1}{25000} = 4 \cdot 10^{-5}$.

(*i*) You are given a laptop that has already been used for $4000 = 4 \cdot 10^3$ hours. What is the probability that you will be able to use it for another $10000 = 10^4$ hours? [3 marks]

(*ii*) The laptop keeps getting passed on, from one owner to another. Each owner uses this laptop for $10000 = 10^4$ hours. This continues until the laptop breaks down. What is the probability that the laptop breaks down for the n^{th} owner, $n \geq 1$? [4 marks]

(*iii*) Let R be a random variable for the number of laptop owners until it breaks down. Show that R is a geometric random variable, and give the value of its parameter p. [3 marks]

(*iv*) Consider now two different laptops with lifetimes X_1, X_2, which are two independent exponential random variables with rates $\lambda_1 = 4 \cdot 10^{-5}$ and $\lambda_2 = 8 \cdot 10^{-5}$. What is the expected time until the first of the two laptops breaks down? [4 marks]

(*b*) We know that a laptop battery breaks down from new after T charging cycles, where T is a geometric random variable with parameter $p \in (0, 1)$.

(*i*) What is $\mathbb{E}[T]$? Find an unbiased estimator for $\mathbb{E}[T]$. [2 marks]

(*ii*) Find an unbiased estimator for p and interpret the result. [4 marks]