

3 Complexity Theory (ad260)

- (a) If A and B are decision problems, we write $A \leq_L B$ to denote that A is reducible to B by means of a *logarithmic-space* reduction. Give a precise definition of such a reduction. [2 marks]
- (b) For decision problems A , B and C , show that if $A \leq_L B$ and $B \leq_L C$, we have $A \leq_L C$. [5 marks]
- (c) For each of the four complexity classes P, NP, NL and co-NP, give an example of a problem that is *complete* for the complexity class under logarithmic-space reductions. You do not need to prove the completeness. [4 marks]
- (d) For each pair of problems A and B from your answers to part (c) above, state whether or not $A \leq_L B$, or if this is unknown. Where it is unknown, state any consequences about the inclusion of complexity classes that would follow from $A \leq_L B$. [9 marks]