5 Computation Theory (amp12)

Given a partial function $f : \mathbb{N} \rightharpoonup \mathbb{N}$, let $D(f)$ denote the set of natural numbers at which f is defined: $D(f) = \{x \in \mathbb{N} \mid f(x){\downarrow}\}$; and let $I(f)$ be the set of natural numbers that are values of f where it is defined: $I(f) = \{y \in \mathbb{N} \mid \text{for some } x,\ f(x) = y\}$.

Prove or disprove the following statements, clearly stating any results about register machine computable functions and partial recursive functions that you use.

(*a*) Every subset $S \subseteq \mathbb{N}$ is equal to $I(f)$ for some register machine computable partial function f. [4 marks]

(*b*) If f is register machine computable, then $I(f)$ is equal to $D(g)$ for some partial recursive function g. [7 marks]

(*c*) If f is register machine computable, then $D(f)$ is equal to $I(g)$ for some *total* recursive function g. [4 marks]

(*d*) If g is a partial recursive function, then $D(g)$ is equal to $I(f)$ for some register machine computable partial function f. [5 marks]