

9 Logic and Proof (lp15)

- (a) You have to write a program to classify given propositional formulas as unsatisfiable, valid or neither. You have the option of using either BDDs or DPLL for this. For each option, sketch how to carry out the classification and comment briefly on its advantages (e.g. from the standpoint of performance or space) compared with the alternative. [4 marks]
- (b) It is possible to modify DPLL such that, whenever the empty clause is encountered within the search, a subset of the current variable assignments are identified that contradict the original set of clauses. These assignments are then used to create a new clause. For example, if  $P$  true,  $Q$  false,  $S$  false contradict the original clauses, then the new clause  $\{\neg P, Q, S\}$  is added to the others.
- (i) In what sense is adding this clause logically correct? [2 marks]
- (ii) How is this new clause likely to affect the operation of DPLL? [2 marks]
- (c) For each of the following formulas, either exhibit a formal proof in the free-variable tableau calculus, or exhibit a falsifying interpretation ( $a$  is a constant):
- (i)  $[\neg P(a) \wedge \forall x (\neg P(f(x)) \rightarrow P(x))] \rightarrow \exists x (P(x) \wedge \neg P(f(x)))$  [6 marks]
- (ii)  $(\forall x P(x)) \wedge (\forall x Q(x)) \vee \exists x (P(x) \rightarrow \neg Q(x))$  [6 marks]