

7 Discrete Mathematics (mpf23)

(a) Without using the Fundamental Theorem of Arithmetic, prove that for all positive integers a and b ,

(i) if $\gcd(a, b) = 1$ then, for all integers n , $(a \mid n \wedge b \mid n) \Rightarrow (a \cdot b) \mid n$; [6 marks]

(ii) if $(a \mid n \wedge b \mid n) \Rightarrow (a \cdot b) \mid n$, for all integers n , then $\gcd(a, b) = 1$. [6 marks]

(b) Let U be a set. Prove that, for all sets A, B, C in $\mathcal{P}(U)$,

$$(A \cap B) \cup (A^c \cap C) \cup (B \cap C) = (A \cap B) \cup (A^c \cap C)$$

[4 marks]

(c) Say whether the following statement is true or false, and prove your claim.

For all sets A and subsets $S \subseteq A$, there exists a function $f : A \rightarrow S$ such that, for all $s \in S$, $f(s) = s$. [4 marks]