COMPUTER SCIENCE TRIPOS Part IB - 2023 - Paper 4

2 Compiler Construction (jdy22)

Here is an OCaml definition of the Ackermann function, ack:

```
let rec ack m n =
if m = 0 then n+1
else if n = 0 then ack (m-1) 1
else ack (m-1) (ack m (n-1))
```

- (a) You would like to run ack on an old system with limited stack space and no support for closures, and decide to rewrite it in stages.
 - (i) Rewrite the ack function to produce a function ack_cps in continuation-passing style so that the function

```
let ack_1 m n = ack_{cps} m n (fun x -> x)
```

produces the same results as the function ack.

[5 marks]

(ii) Eliminate higher-order functions from your answer to Part (a)(i) by rewriting ack_cps as a function ack_cps_dfn in defunctionalized form so that the function

let
$$ack_2 m n = ack_cps_dfn m n ID$$

produces the same results as the function ack.

[5 marks]

- (iii) Convert your answer from Part (a)(ii) to a function ack_cps_dfn_list that uses standard lists rather than custom data types [5 marks]
- (b) Briefly comment on the way that ack and the transformed implementations in Parts (a)(i), (a)(ii) and (a)(iii) use memory, making reference to the stack and heap. [5 marks]