

8 Logic and Proof (mj201)

Which of the following formulas is valid? Prove them using resolution or find a counterexample. If a counterexample exists, explain how resolution is blocked from producing a spurious proof.

(a) $(\forall x [\neg Q(x, x) \rightarrow P(x)]) \rightarrow \forall x [P(x) \vee \exists y Q(x, y)]$ [7 marks]

(b) $[\exists x P(x)] \rightarrow \exists x P(f(x))$ [6 marks]

(c) $\exists z \forall w [(\forall x [P(x) \rightarrow \exists y Q(x, y)]) \rightarrow (P(w) \rightarrow Q(w, z))]$ [7 marks]