8 Further Graphics (aco41)

(*a*) We are given a scene with a point light source at a point $\mathbf{p}$ that emits light of radiance L in all directions, and a plane with equation $\mathbf{n}^T(\mathbf{x}-\mathbf{c}) = 0$ and BRDF f_r. We send a ray from the camera sensor with origin $\mathbf{o}$ and direction $\mathbf{d}$ such that each point on the ray is $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$ for $t > 0$.

(*i*) What is the intersection point of the ray and the plane? [2 marks]

(*ii*) What is the incoming light direction? [1 mark]

(*iii*) What is the direction of the outgoing reflected light towards the sensor origin $\mathbf{o}$? [1 mark]

(*iv*) What is the radiance of the light reflected towards the sensor origin $\mathbf{o}$? [2 marks]

(*b*) A surface represented by an implicit function $f(\mathbf{x})$ is transformed by linear blend skinning such that each point $\mathbf{x}$ is mapped to $\mathbf{x}'$. There are two bones with transformation matrices $\mathbf{T}_1$ and $\mathbf{T}_2$ and corresponding blending weights $w_1(\mathbf{x})$ and $w_2(\mathbf{x}) = 1 - w_1(\mathbf{x})$. In addition, we are given $s_1(\mathbf{x}') = w_1(\mathbf{x})$ and $s_2(\mathbf{x}') = w_2(\mathbf{x})$.

(*i*) Write the expression for $\mathbf{x}'$. [1 mark]

(*ii*) Simplify the expression for $\mathbf{x}'$ assuming no rotations are stored on the bones. Modify the expression further for the case where the weights are not summing up to 1. [2 marks]

(*iii*) What is the implicit function of the transformed surface assuming all blended rotations are invertible? Simplify as much as possible. [3 marks]

(*iv*) What is the implicit function of the transformed surface if there are no rotations stored on the bones? Simplify as much as possible. [2 marks]

(*c*) In this question, we will think about how quaternions represent spatial rotations.

(*i*) Write the form of a general quaternion and the form of a quaternion representing rotations. [2 marks]

(*ii*) Prove that a quaternion $\mathbf{q}$ and $-\mathbf{q}$ represents the same rotation. [2 marks]

(*iii*) Given a quaternion $\mathbf{q}$ with no real part, what is $\mathbf{qq}$? [2 marks]