

13 Types (nk480)

(a) Derive the following entailments with the natural deduction system for classical logic.

(i) Show $\neg(A \vee B); \cdot \vdash \neg A$ true. [5 marks]

(ii) Show $\cdot; \neg A \vee \neg B \vdash A$ true. [5 marks]

(b) (i) Using $fold : \forall a. a \rightarrow (X \rightarrow a \rightarrow a) \rightarrow \text{List}_X \rightarrow a$,
 $cons : X \rightarrow \text{List}_X \rightarrow \text{List}_X$ and $nil : \text{List}_X$, write a System F function which
 appends two lists. [1 mark]

(ii) Give an OCaml data structure corresponding to the following Church encoding:

$$\forall a. a \rightarrow (a \rightarrow X \rightarrow a \rightarrow a) \rightarrow a$$

[2 marks]

(iii) Give a System F term which converts an element t of the type in part (ii) of this question into a list with the same elements. [3 marks]

(c) Consider the following two Agda proofs:

$$\begin{array}{ll} \text{unitl} & : \quad \forall x \rightarrow 0 + x \equiv x \\ \text{unitl } x & = \text{refl}(x) \end{array} \qquad \begin{array}{ll} \text{unitr} & : \quad \forall x \rightarrow x + 0 \equiv x \\ \text{unitr } 0 & = \text{refl}(0) \\ \text{unitr } (s \ n) & = \text{cong } s \ (\text{unitr } n) \end{array}$$

Explain why they are different.

[4 marks]