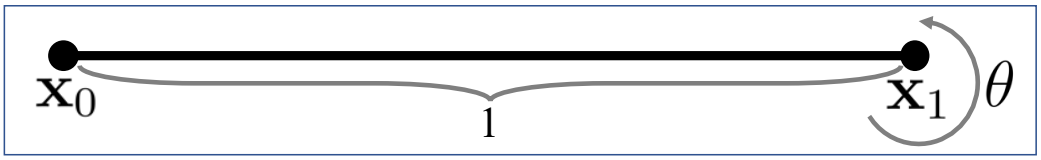
COMPUTER SCIENCE TRIPOS Part IB – 2024 – Paper 7

7 Further Graphics (aco41)

(*a*) You are given a unit length rod as in the figure. There is no rotation at one end of the rod at point $\mathbf{x}_0$ and a rotation around the plane normal $\mathbf{n}$ is defined at the other end $\mathbf{x}_1$. This rotation is interpolated along the rod. The interpolation weight for $\mathbf{x}$ on the rod is $||\mathbf{x} - \mathbf{x}_i||$ for the rotation at $\mathbf{x}$, $i = 0, 1$.



(*i*) Write the rotation in quaternion form at $\mathbf{x}$ given a rotation of $\theta \leq \pi$ at $\mathbf{x}_1$ assuming shortest path interpolation of rotations in SO(3). [3 marks]

(*ii*) At $\mathbf{x}_1$ at time $t = 0$ there is no rotation and at $t = 1$ the rotation angle is $\pi/2$. Write the quaternion at $\mathbf{x}$ at any time in $[0, 1]$ assuming the shortest path interpolation in SO(3) over time and over the rod. [3 marks]

(*iii*) Answer (*ii*), this time assuming linear blending of quaternions along the rod and the shortest path interpolation in SO(3) over time. [3 marks]

(*iv*) Write an expression for the norm of the quaternion in (*iii*). [3 marks]

(*b*) Given a triangle in 3D with vertex locations $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$,

(*i*) determine a condition on the vertices for the triangle to define a valid plane, [1 mark]

(*ii*) define a parametric form $\mathbf{p}(u, v)$ for the plane of the triangle assuming it defines a valid plane, [2 marks]

(*iii*) write an expression for the normal of the plane, [1 mark]

(*iv*) write the steps of an algorithm to find the closest point of a point $\mathbf{x}$ in space on the triangle. [4 marks]