## 11   Quantum Computing (sjh227)
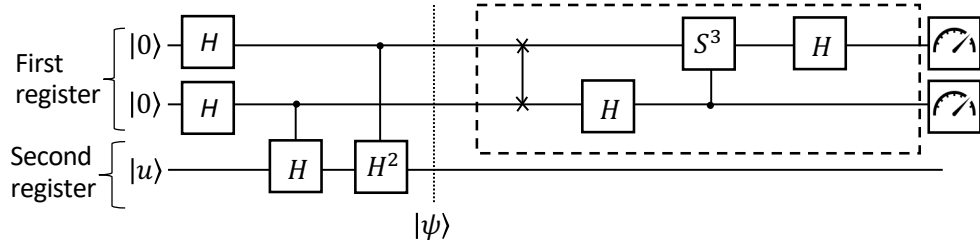


(a)   The figure shows the circuit for quantum phase estimation of a Hadamard gate. What is the function of the sub-circuit shown in the box marked with the dashed line, and to how many bits of precision is the estimate of the phase given?

[2 marks]

(b)   The Hadamard gate has matrix $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$. What are its eigenvectors and corresponding eigenvalues? Express each eigenvector as a quantum state (that is, as superposition of computational basis states). [5 marks]

(c)   Simplify the circuit in the figure such that when the initial state of the first register is $|00\rangle$ as specified, the top wire only involves a swap gate and a measurement. [6 marks]

(d)   Quantum phase estimation is performed using the circuit given in the figure with $|u\rangle = a|0\rangle + b|1\rangle$. Express the three-qubit state $|\psi\rangle$ in terms of $a$ and $b$. Verify that if $|u\rangle$ is a correctly normalised quantum state then so is $|\psi\rangle$. [7 marks]