COMPUTER SCIENCE TRIPOS Part IA − 2025 − Paper 1

5   Introduction to Probability (mj201+tms41)

For each of the three distributions, state the probability mass function and expectation:

(a) Binomial Distribution,   [2 marks]

(b) Poisson Distribution,   [2 marks]

(c) Hypergeometric Distribution.   [2 marks]

Assume we want to load one container with 5 packets with weights X_i, $1 \leq i \leq 5$, which are independent exponential random variables with parameter $\lambda = 1/2$.

(d) Let $X := \sum_{i=1}^{5} X_i$ be the sum of weights. Compute the expectation $\mathbf{E}\,[\,X\,]$ and variance $\mathbf{V}\,[\,X\,]$.   [3 marks]

We want to find a weight capacity w of the container such that with probability at least 0.95 we can load all 5 packets onto the container.

(e) Using Markov's inequality, find a solution for w.   [3 marks]

(f) Using Chebyshev's inequality, find a solution for w.   [4 marks]

(g) Using the Central Limit Theorem, find a solution for w.
     Remark: You should **not** to give a numerical result, but your answer may involve the function $\Phi(.)$.   [4 marks]