

COMPUTER SCIENCE TRIPOS Part IA – 2025 – Paper 1

7 Algorithms 1 (jkf21)

- (a) Find asymptotically tight lower and upper bounds for the following recurrence relation and explain your answer.

$$T(1) = 1$$

$$T(n) = 5T(n/5) + n^3$$

[7 marks]

- (b) Find an asymptotically tight upper bound for the recurrence relation

$$T(1) = 1$$

$$T(n) = T(n - 1) + \lg n$$

where \lg denotes base-2 logarithms. Explain your answer.

[3 marks]

- (c) Show that the recurrence relation

$$T(1) = 1$$

$$T(n) = T(n/a) + \lg n$$

is in $\omega(\lg n)$ and $O(n)$, where $a > 1$ is a real-valued parameter and \lg denotes base-2 logarithms. Is $T(n) \in o(\lg^2 n)$? Explain your answer.

[7 marks]

- (d) Let $T(n)$ be the recurrence relation defined by

$$T(1) = 1$$

$$T(n) = T(n/5) + T(3n/5) + kn$$

where $k > 0 \in \mathbb{R}$. Is $T(n) \in O(n)$? Justify your answer.

[3 marks]