

1 Complexity Theory (tg508)

Let **SSP** be the Subset Sum Problem, defined as follows.

Instance: A finite set of integers S and a target integer t .

Problem: Does there exist a subset $S' \subseteq S$ such that $\sum_{s \in S'} s = t$?

- (a) Provide two definitions of the class NP: one in terms of non-determinism and the other in terms of verification. Prove that the two notions are equivalent. [6 marks]
- (b) Show that **SSP** $\in$ NP. [2 marks]
- (c) Prove that **SSP** is NP-complete. (*Hint: to establish hardness, you can reduce from the NP-complete 3D Matching Problem, defined as follows. Given disjoint sets X, Y, Z , each of size n , and a collection of triples $T \subseteq X \times Y \times Z$, determine whether there exists a subset $M \subseteq T$ of size n such that each element of X, Y , and Z appears in exactly one triple of M .*) [8 marks]
- (d) Suppose that one finds a non-deterministic logspace algorithm for **SSP**. What would this imply about the NP vs co-NP problem? [4 marks]