COMPUTER SCIENCE TRIPOS Part II – 2025 – Paper 8

11 Quantum Computing (pm830)

Quantum gate teleportation. Consider a system with two data qubits (D_1, D_2) and two Bell state qubits (B_1, B_2). D_1 and D_2 are initialized to $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ with a, b, c, d being suitably normalized. B_1 and B_2 are initialized to $(1/\sqrt{2})(|01\rangle + |10\rangle)$.

(a) Give a sequence of gates which can transform a state $|00\rangle$ to $(1/\sqrt{2})(|01\rangle + |10\rangle)$. [4 marks]

(b) Given a quantum state $\alpha|0\rangle + \beta|1\rangle$ where α and β are suitably normalized, let's measure it in the X basis (i.e., in the $|+\rangle$, $|-\rangle$ basis). What is the probability of measuring the state $|+\rangle$? What is the probability of measuring the state $|-\rangle$? [2 marks]

(c) The following operations are executed in sequence.

1. CNOT D_1, B_1 // Controlled NOT with control D_1 and target B_1
2. CNOT B_2, D_2
3. x = Measure B_1 in Z basis
4. y = Measure B_2 in X basis
5. If x is 0, apply X gate on D_2. Else apply I gate on D_2.
6. If y is 0 (i.e., the state is $|+\rangle$, apply I gate on D_1. Else apply Z gate on D_1.

Let's analyze how the state of the 4-qubit system changes as we execute the operations above. What is the state of the system after steps 1 and 2? For all states, use the ordering convention $|D_1 D_2 B_1 B_2\rangle$. [4 marks]

(d) Given particular values for x and y, what is the state of the system after steps 3 and 4? [4 marks]

(e) Prove that the overall effect of the sequence 1-6 is to apply a CNOT gate with D_1 as control and D_2 as target, up to global phase. [4 marks]

(f) Suppose B_1 and B_2 were initialized to the state $(1/\sqrt{2})(|00\rangle + |11\rangle)$, how should steps 5 and 6 be modified to realize the CNOT gate between D_1 and D_2? [2 marks]