Strategic Principles in the Design of Isabelle

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Proof Assistants: A Strategic View

Strength over the long term

- automation: essential in an interactive tool
- flexibility: for the differing needs of users
  - control over syntax
  - a choice of logical formalisms (logical framework!)
  - a toolkit for proof strategies
- soundness needs a small trusted kernel

Automation & Flexibility. . . How?

- higher-order syntax
- logical variables and unification
- search primitives based on lazy lists

(Can logical frameworks really work?)

a sort of higher-order Prolog (like Dale Miller's λProlog)

Higher-Order Syntax: A Must!

Flexibility: users can define new variable binders

$$\text{least } n.\ P(n) \qquad \{x \in A \mid P(x)\} \qquad \bigcup_{x \in A} B(x)$$

$$\text{case } l \text{ of } [] \Rightarrow z \mid x\#l' \Rightarrow f(x, l')$$

Doesn’t require higher-order logic

Alternatives?? Combinators or auxiliary functions

Logical Variables

- don't know subterms can be left unspecified . . .
- . . . until unification completes them
- helpful for proof procedures
- declarative representation of rules

rare in higher-order proof tools

Declarative Rules

Define the quantifier $\forall_{x \in A} P(x)$ to be $\forall x \, [x \in A \rightarrow P(x)]$

Derive the rule

$$\frac{\forall_{x \in A} P(x) \qquad a \in A}{P(a)}$$

Can be displayed and transformed and combined (resolution!)

Alternative representations: code, or higher-order formula

Higher-Order + Logical Variables = ?

Higher-order unification (Huet, 1975)

In the worst case. . .

- infinitely many unifiers
- semi-decidable
- complicated algorithm

Pattern unification handles the easy cases (Miller's L_λ)

Tactics Based on Lazy Lists

Tactics describe the search space

- proof state $\longrightarrow$ list of proof states
- result is a lazy list

Tacticals explore the search space

- tactic $\longrightarrow$ tactic
- strategies: depth-first, best-first, iterative deepening, . . .

Strategies are easily combined

Automation in Predicate Logic

Tableaux-style provers for intuitionistic and classical FOL

The MESON proof procedure (world's slowest!)

A generic classical reasoner (here, in ZF set theory):

$$C \neq \emptyset \to \bigcap_{x \in C} [A(x) \cap B(x)] = (\bigcap_{x \in C} A(x)) \cap (\bigcap_{x \in C} B(x))$$

1/2 second on Pentium

More Automation: Inductive Definitions

To formalize

- operational semantics: languages, type theories, . . .
- proof systems
- security

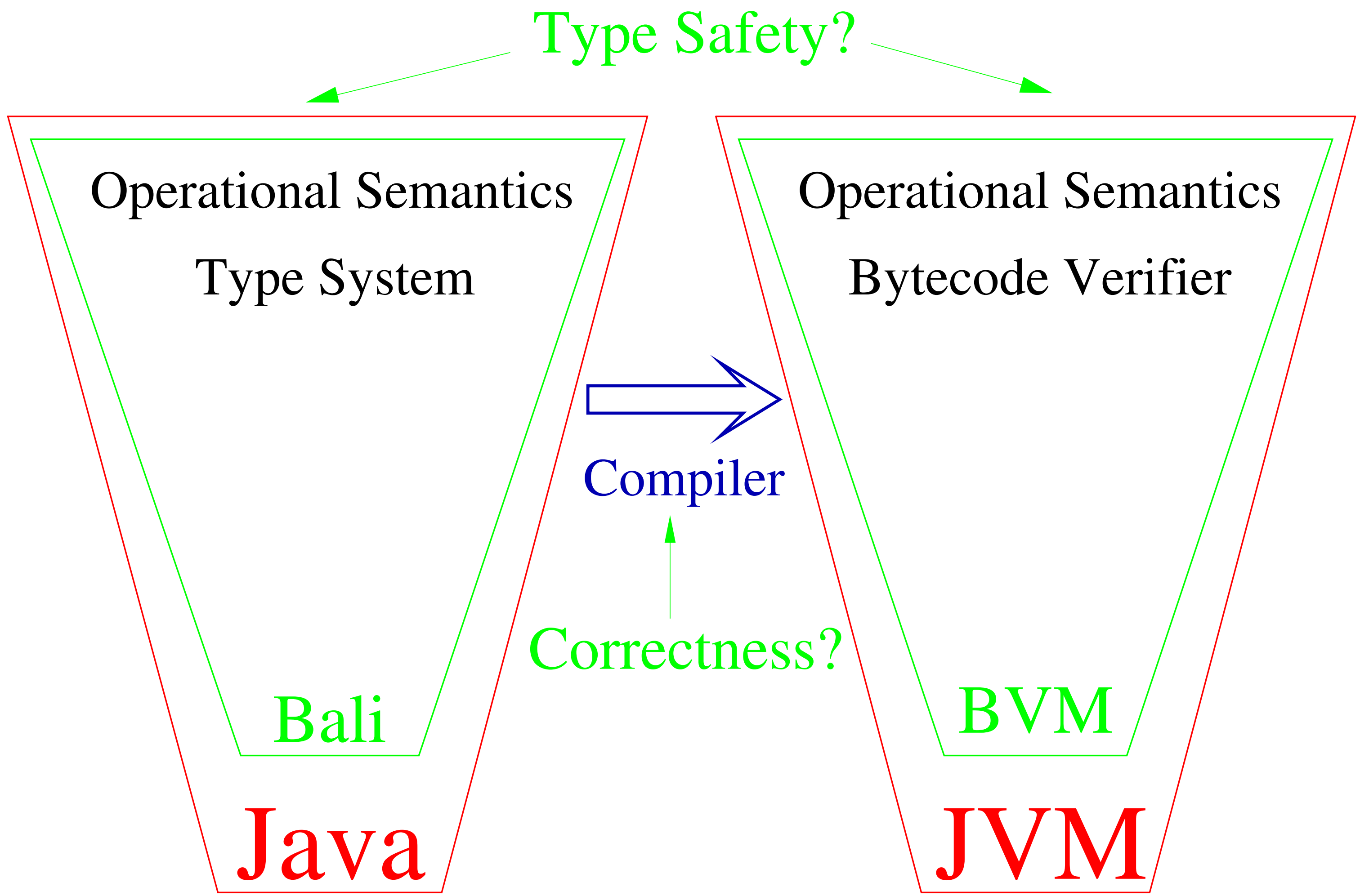
Induction rules proved, not assumed

Proofs generated using tactics & tacticals

Keep the trusted kernel small

Some Applications

- temporal reasoning: UNITY, TLA, . . . (TUM and Cambridge)
- combinations of non-classical logics (MPI-Saarbrücken)
- verification of cryptographic protocols (Cambridge)
- Java type safety (TUM)


Type Safety?
Operational Semantics
Type System
Bali
Java
Compiler
Correctness?
Operational Semantics
Bytecode Verifier
BVM
JVM

Bali and BVM

Bali: a large subset of Java

- class, interface, field & method
- inheritance, overriding, & hiding
- overloading, dynamic binding, exceptions. . .

Bali Virtual Machine

- OO concepts (as above)
- integers & arrays
- predefined exceptions

Bytecode Verifier BVM

Cornelia Pusch: Isabelle proof of

$$ok(bytecode) \Rightarrow \text{no runtime type error}$$

Bali	Formalization:	1200 lines	5 weeks
	Proof of type safety:	2400 lines	10 weeks
BVM	Formalization BVM:	1100 lines	7 weeks
	Formalization BV:	600 lines	5 weeks
	Proof of type safety:	3000 lines	8 weeks

Can Cryptography Make Networks Secure?

Goals of security protocols:

- Authenticity: who sent this message?
- Secrecy: who can receive my message?

Threats:

- Active attacker
- Careless & compromised agents . . . NO code-breaking

The Needham-Schroeder Protocol (1978)

$$1.\quad A \longrightarrow B : \{Na, A\}_{Kb}$$

Alice sends Bob an encrypted nonce

$$2.\quad B \longrightarrow A : \{Na, Nb\}_{Ka}$$

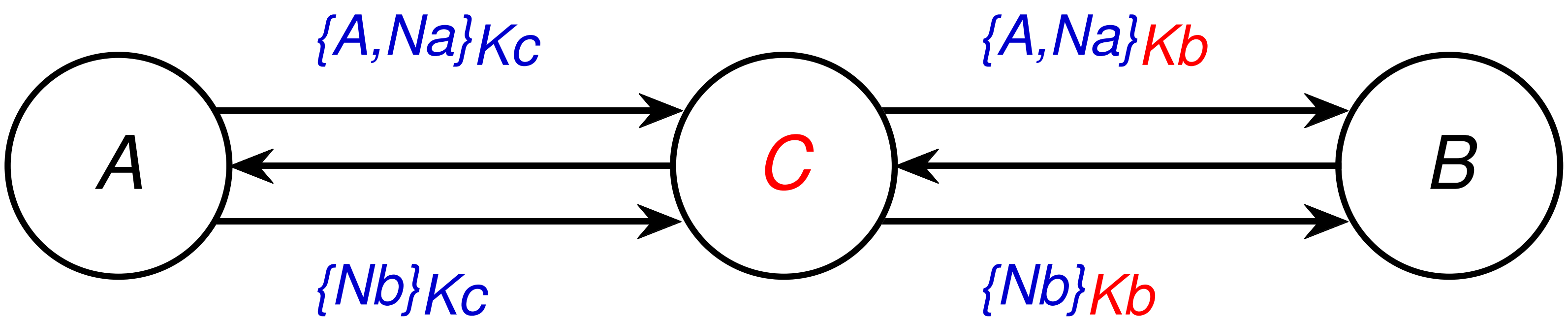
Bob returns Na with a nonce of his own

$$3.\quad A \longrightarrow B : \{Nb\}_{Kb}$$

Alice returns Bob's nonce

A Middle-Person Attack (1995)

Villain Charlie can masquerade as Alice to Bob



Gavin Lowe found this attack 17 years later!

Verification Methods

- Logics of belief (BAN, 1989)
  - Allows short, abstract proofs but misses many flaws
- State enumeration
  - Automatically finds attacks but requires strong assumptions
- Inductive protocol verification
  - Trace model of agents
  - proofs mechanized using Isabelle/HOL

Protocol Verification: Results

- industrial protocols analyzed (TLS, Kerberos, . . .)
- minutes CPU time, weeks human time per protocol
- the power of
  - inductive definitions
  - the simplifier
  - the classical reasoner
- substantial proofs found automatically

Conclusions

- logical frameworks can be practical
- lazy lists give the needed flexibility
- higher-order syntax can be combined with logical variables
- ATP techniques can be used in an interactive tool

. . . plus a lot of hard work to make it go!