

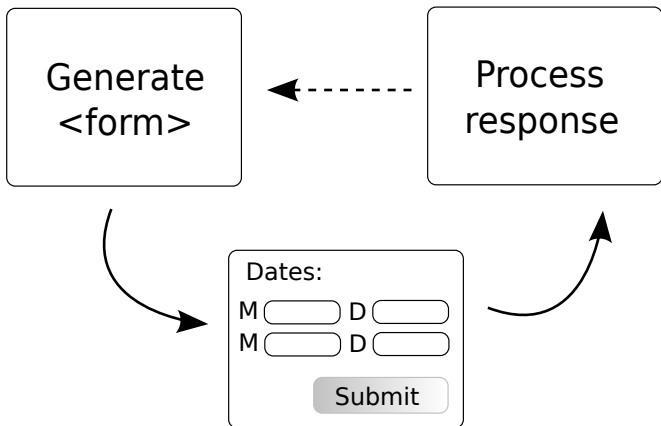
The ML module system

Abstractions for programming in the large

Jeremy Yallop

28 November 2013

Programming with forms



Formlets

```
let date_formlet : date formlet =  
  formlet  
    <div>  
      Month: {input_int  $\Rightarrow$  month}  
      Day: {input_int  $\Rightarrow$  day}  
    </div>  
  yields  
    (make_date month day)
```

Formlets

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let date_formlet : date formlet =  
  formlet  
    <div>  
      Month: {input_int ⇒ month}  
      Day: {input_int ⇒ day}  
    </div>  
  yields  
    (make_date month day)
```

```
let date_formlet : date formlet =  
  pure (fun (), month, (), day, ()) → make_date month day)  
  ⊗ (tag "div" []  
    (pure (fun () month () day () → ((), month, (), day, ()))  
      ⊗ text "Month: " ⊗ input_int  
      ⊗ text "Day: " ⊗ input_int ⊗ text "\n "))
```

Formlets

```
let travel_formlet : (string * date * date) formlet =  
  formlet  
    <#> Name: {input ⇒ name}  
      <div>  
        Arrive: {date_formlet ⇒ arrive}  
        Depart: {date_formlet ⇒ depart}  
      </div>  
      {submit "Submit"}  
    </#>  
  yields (name, arrive, depart)
```

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let travel_formlet : (string * date * date) formlet =  
  formlet  
    <#> Name: {input ⇒ name}  
      <div>  
        Arrive: {date_formlet ⇒ arrive}  
        Depart: {date_formlet ⇒ depart}  
      </div>  
      {submit "Submit"}  
    </#>  
  yields (name, arrive, depart)
```

```
let travel_formlet : (string × date × date) formlet =  
  pure (fun (), name, (), ((), arrive, (), depart, ()), (), (), ()) →  
    (name, arrive, depart))  
  ⊗ (pure (fun () name () ((), arrive, (), depart, ()) () () () →  
    ((), name, (), ((), arrive, (), depart, ()), (), (), ()))  
    ⊗ text "\n  Name: " ⊗ input ⊗ text "\n "  
    ⊗ (tag "div" []  
      (pure (fun () arrive () depart () → ((), arrive, (), depart, ()))  
        ⊗ text "\n    Arrive: " ⊗ date_formlet  
        ⊗ text "\n    Depart: " ⊗ date_formlet ⊗ text "\n ")  
      ⊗ text "\n  " ⊗ xml (submit "Submit") ⊗ text "\n ")
```

Desugaring formlets

$$\begin{aligned}
 \llbracket r \rrbracket &= r^* \\
 \llbracket \text{formlet } q \text{ yields } e \rrbracket &= \text{pure } (\text{fun } q^\dagger \rightarrow \llbracket e \rrbracket) \otimes q^\circ \\
 s^* &= \text{xml_text } s \\
 \{e\}^* &= \llbracket e \rrbracket \\
 (\langle t \text{ ats} \rangle m_1 \dots m_k \langle /t \rangle)^* &= \text{xml_tag } t \text{ ats } (\langle \# \rangle m_1 \dots m_k \langle / \# \rangle)^* \\
 (\langle \# \rangle m_1 \dots m_k \langle / \# \rangle)^* &= m_1^* @ \dots @ m_k^* \\
 s^\circ &= \text{text } s \\
 \{e\}^\circ &= \text{xml } \llbracket e \rrbracket \\
 \{f \Rightarrow p\}^\circ &= \llbracket f \rrbracket \\
 (\langle t \text{ ats} \rangle n_1 \dots n_k \langle /t \rangle)^\circ &= \text{tag } t \text{ ats } (\langle \# \rangle n_1 \dots n_k \langle / \# \rangle)^\circ \\
 (\langle \# \rangle n_1 \dots n_k \langle / \# \rangle)^\circ &= \text{pure } (\text{fun } n_1^\dagger \dots n_k^\dagger \rightarrow (n_1^\dagger, \dots, n_k^\dagger)) \\
 &\quad \otimes n_1^\circ \dots \otimes n_k^\circ \\
 s^\dagger &= () \\
 \{e\}^\dagger &= () \\
 \{f \Rightarrow p\}^\dagger &= p \\
 (\langle t \text{ ats} \rangle n_1 \dots n_k \langle /t \rangle)^\dagger &= (n_1^\dagger, \dots, n_k^\dagger) \\
 (\langle \# \rangle n_1 \dots n_k \langle / \# \rangle)^\dagger &= (n_1^\dagger, \dots, n_k^\dagger)
 \end{aligned}$$

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 (\langle \# \rangle n_1 \dots n_k \langle / \# \rangle)^\circ &= \text{pure } (\text{fun } n_1^\dagger \dots n_k^\dagger \rightarrow (n_1^\dagger, \dots, n_k^\dagger)) \\
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 (\langle \# \rangle n_1 \dots n_k \langle / \# \rangle)^\dagger &= (n_1^\dagger, \dots, n_k^\dagger)
 \end{aligned}$$

Optimising formlets

The `input_int` formlet:

```
let input_int : int formlet =  
  formlet <#>{input ⇒ i}</#>  
  yields (int_of_string i)
```

Optimising formlets

The `input_int` formlet:

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let input_int : int formlet =  
  formlet <#>{input  $\Rightarrow$  i}</#>  
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`input_int` desugared:

```
pure (fun i  $\rightarrow$  int_of_string i)  $\otimes$  (pure (fun i  $\rightarrow$  i)  $\otimes$  input)
```

Optimising formlets

The `input_int` formlet:

```
let input_int : int formlet =  
  formlet <#>{input ⇒ i}</#>  
  yields (int_of_string i)
```

`input_int` desugared:

$$\text{pure } (\text{fun } i \rightarrow \text{int_of_string } i) \otimes (\text{pure } (\text{fun } i \rightarrow i) \otimes \text{input})$$

`input_int` simplified:

$$\text{pure int_of_string} \otimes \text{input}$$

Applicative functor laws

Laws:

$\text{pure } f \otimes \text{pure } x$	$\equiv$	$\text{pure } (f \ x)$	homomorphism
$\text{pure } \text{id} \otimes u$	$\equiv$	u	identity
$\text{pure } (\circ) \otimes u \otimes v \otimes w$	$\equiv$	$u \otimes (v \otimes w)$	composition
$u \otimes \text{pure } x$	$\equiv$	$\text{pure } ((>) \ x) \otimes u$	interchange

where

id	$=$	$\text{fun } x \rightarrow x$
$(\circ)$	$=$	$\text{fun } f \ g \ x \rightarrow f \ (g \ x)$
$(>)$	$=$	$\text{fun } x \ f \rightarrow f \ x$

Normal form:

$$\text{pure } f \otimes u_1 \otimes u_2 \otimes \dots \otimes u_n$$

Plan

Aim: justify post-desugaring simplification by showing that the idiom laws hold for formlets

Approach: use the module system to construct the formlet definition from (trivial) components

Benefits: Easy to reason about; easy to substitute components later on without throwing everything away

Module primer (I)

Module definitions

```
module M : sig = mexp
```

Module expressions

```
struct decls end
```

```
M1.M2. ... .Mn
```

Using modules

```
M.x    M.t    M.N
```

```
include mexp
```

Signature definitions

```
module type S = sigexp
```

Signature expressions

```
sig decls end
```

```
M1.M2. ... .Mn.S
```

Using signatures

```
include sexp
```

```
module type ASSOC_LIST =
```

```
sig
```

```
  type  $\alpha$  t
```

```
  type key
```

```
  include Std.MAPPABLE
```

```
  val length : t  $\rightarrow$  int
```

```
  val assoc : key  $\rightarrow$   $\alpha$  list  $\rightarrow$   $\alpha$  option  
end
```

```
module Assoc_list : ASSOC_LIST =
```

```
struct
```

```
  type key = String.t
```

```
  type  $\alpha$  t = (key  $\times$   $\alpha$ ) list
```

```
  include Std.List
```

```
  let rec assoc v l = match l with
```

```
    | []  $\rightarrow$  None
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```
    | x :: xs when v = x  $\rightarrow$  Some x
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    | x :: xs  $\rightarrow$  assoc v xs
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```
end
```

The idiom interface

```
module type IDIOM =  
sig  
  type  $\alpha$  t  
  val pure :  $\alpha \rightarrow \alpha$  t  
  val ( $\otimes$ ) : ( $\alpha \rightarrow \beta$ ) t  $\rightarrow$   $\alpha$  t  $\rightarrow$   $\beta$  t  
end
```

Formlets, monolithically

```
module type FORMLET =  
sig  
  include IDIOM  
  val xml : xml → unit t  
  val text : string → unit t  
  val tag : tag → attrs →  $\alpha$  t →  $\alpha$  t  
  val input : string t  
  val run :  $\alpha$  t → xml × (env →  $\alpha$ )  
end
```

```
module Formlet : FORMLET =  
struct  
  type  $\alpha$  t =  
    int → (xml × (env →  $\alpha$ ) × int)  
  
  let pure x i = ([], const x, i)  
  
  let ( $\otimes$ ) f p i =  
    let ( $x_1$ , g, i) = f i in  
    let ( $x_2$ , q, i) = p i in  
    ( $x_1$  @  $x_2$ , (fun env → g env (q env)), i)  
  
  ...  
  
  let run c = let (x, f, _) = c 0 in (x, f)  
end
```

The environment idiom

```
module type ENVIRONMENT =  
sig  
  include IDIOM  
  type env = (string × string) list  
  val lookup : string → string t  
  val run :  $\alpha$  t → env →  $\alpha$   
end
```

```
module Environment : ENVIRONMENT =  
struct  
  type env = (string × string) list  
  type  $\alpha$  t = env →  $\alpha$   
  let pure v e = v  
  let ( $\otimes$ ) f p e = f e (p e)  
  let lookup = List.assoc  
  let run v = v  
end
```

The environment idiom: laws

We want to show that the homomorphism law

$$\text{pure } f \otimes \text{pure } x \quad \equiv \quad \text{pure } (f \ v)$$

holds for the environment idiom.

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$$\begin{aligned} & \text{pure } f \otimes \text{pure } x \\ \equiv & \quad (\textit{definition of } \otimes \textit{ and pure}) \\ & (\text{fun } f \text{ p e} \rightarrow f \text{ e (p e)}) (\text{fun } _ \rightarrow f) (\text{fun } _ \rightarrow x) \end{aligned}$$

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Let's try proving the identity law:

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$$(\text{fun } f \text{ p } e \rightarrow f \text{ e } (p \text{ e})) (\text{fun } _ \rightarrow \text{id}) u$$

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$$\equiv \text{ (repeated } \beta\text{-reduction)}$$

$$\text{fun } e \rightarrow \text{id } (u \text{ e})$$

$$\equiv \text{ (definition of id)}$$

$$\text{fun } e \rightarrow u \text{ e}$$

$$\equiv \text{ (}\eta\text{-contraction)}$$

$$u$$

The name generation idiom

```
module type NAMER =  
sig  
  include IDIOM  
  val next_name : string t  
  val run :  $\alpha$  t  $\rightarrow$   $\alpha$   
end
```

```
module Namer : NAMER =  
struct  
  type  $\alpha$  t = int  $\rightarrow$   $\alpha$   $\times$  int  
  let pure v i = (v, i)  
  let ( $\otimes$ ) f p i = let (f', i) = f i in  
                   let (p', i) = p i in  
                   (f' p', i)  
  
  let next_name i =  
    ("input_" ^ string_of_int i, i+1)  
  let run v = fst (v 0)  
end
```

The XML accumulation idiom

```
module XMLWRITER =  
sig  
  include IDIOM  
  val text : string → unit t  
  val xml : xml → unit t  
  val tag : tag → attrs →  $\alpha$  t →  $\alpha$  t  
  val run :  $\alpha$  t → xml ×  $\alpha$   
end
```

```
module XmlWriter : XMLWRITER =  
struct  
  type  $\alpha$  t = xml ×  $\alpha$   
  let pure v = ([], v)  
  let ( $\otimes$ ) (x, f) (y, p) = (x @ y, f p)  
  let text x = (xml_text x, ())  
  let xml x = (x, ())  
  let tag t a (x,v) = (xml_tag t a x, v)  
  let run v = v  
end
```

```
type xml = xml_item list  
and tag = string  
and attrs = (string × string) list  
and xml_item
```

```
val tag : tag → attrs → xml → xml  
val text : string → xml
```

Module primer (II)

Module definitions

```
module M (X1 : sig)
  (X2 : ... sig)
  ...
  (Xn : ... sig)
  : sig = mexp
```

Module expressions

```
mexp (mexp)
```

Signature expressions

```
sigexp
with type t1 = type-exp1
and type t2 = type-exp2
and ...
and type tn = type-expn
```

```
module Assoc_list (Key : KEY) :
  ASSOC_LIST
  with type key = Key.t =
  struct
    ...
  end

module String_assoc_list =
  Assoc_list(Std.String)

module Int_assoc_list =
  Assoc_list(Std.Int)
```

Module primer (II)

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```
module M (X1 : sig)
    (X2 : ... sig)
    ...
    (Xn : ... sig)
  : sig = mexp
```

Module expressions

```
mexp (mexp)
```

Signature expressions

```
sigexp
with type t1 = type-exp1
and type t2 = type-exp2
and ...
and type tn = type-expn
```

```
module Assoc_list (Key : KEY) :
    ASSOC_LIST
    with type key = Key.t =
struct
    ...
end
```

```
module String_assoc_list =
    Assoc_list(Std.String)
```

```
module Int_assoc_list =
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```

Module primer (II)

Module definitions

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module M (X1 : sig)
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  ...
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```

Idiom composition

```
module Compose (F : IDIOM) (G : IDIOM) :  
sig  
  include IDIOM with type  $\alpha$  t = ( $\alpha$  G.t) F.t  
  val refine :  $\alpha$  F.t  $\rightarrow$  ( $\alpha$  G.t) F.t  
end  
=  
struct  
  type  $\alpha$  t = ( $\alpha$  G.t) F.t  
  let pure x = F.pure (G.pure x)  
  let ( $\otimes$ ) f x = F.pure ( $\otimes_G$ )  $\otimes_F$  f  $\otimes_F$  x  
  let refine v = (F.pure G.pure)  $\otimes_F$  v  
end
```

Idiom composition laws

We want to show that the homomorphism law

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Formlets, compositionally

```
module type FORMLET =  
sig  
  include IDIOM  
  val xml : xml → unit t  
  val text : string → unit t  
  val tag : tag → attrs →  $\alpha$  t →  $\alpha$  t  
  val input : string t  
  val run :  $\alpha$  t → xml × (env →  $\alpha$ )  
end
```

```
module Formlet : FORMLET =  
struct  
  include Compose (Namer)  
    (Compose (XmlWriter)  
      (Environment))  
  ...  
  let run v =  
    let xml, collector =  
      XmlWriter.run (Namer.run v)  
    in  
      (xml, Environment.run collector)  
end
```

Closing quote

We propose [...] that one begins with a list of difficult design decisions or design decisions which are likely to change. Each module is then designed to hide such a decision from the others. [...] To achieve an efficient implementation we must abandon the assumption that a module is one or more subroutines, and instead allow subroutines and programs to be assembled collections of code from various modules.

(On the criteria to be used in decomposing systems into modules
D. L. Parnas (1972))